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Giant Gravitons and a Correspondence Principle

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Abstract

We propose a correspondence between the physics of certain small charge black holes in $\mathrm{AdS}_k \times \mathrm{S}^l$ and large charge black holes in $\mathrm{AdS}_l \times \mathrm{S}^k$. The curvature singularities of these solutions arise, following Myers and Tafjord, from a condensate of giant gravitons. When the number of condensed giants N_g is much greater than the number of background branes N, we propose that the system has an equivalent description in terms of N giant gravitons condensed in a background created by N_g branes. Our primary evidence is an exact correspondence between gravitational entropy formulae of small and large charge solutions in different dimensions.

1 Introduction

String theory enjoys a number of duality symmetries that relate the physics of apparently different theories in different dimensions. A particularly striking illustration of such a symmetry is the AdS/CFT duality [1], which relates gravity on AdS space to a CFT on the AdS boundary. In this note we propose a correspondence between the physics of large charge black holes in $\mathrm{AdS}_k \times \mathrm{S}^l$ and small charge black holes in $\mathrm{AdS}_l \times \mathrm{S}^k$.

The black holes in question are electrically charged under $U(1)$ gauge fields arising from Kaluza-Klein reduction of 10d or 11d supergravity on a sphere. Myers and Tafjord [2] showed that the curvature singularities of such charged solutions of $\mathrm{AdS}_5 \times \mathrm{S}^5$ gravity can be understood as condensates of giant gravitons [3] on S^5. In Sections 2 and 3 we extend the work of Myers and Tafjord [2] to charged black holes in the four and seven dimensional gauged supergravities. Lifting the solutions to eleven dimensions [4] as asymptotically $\mathrm{AdS}_k \times \mathrm{S}^l$ spaces reveals that their singularities arise from condensation of giant gravitons on S^l. In four and seven dimensions there are four and two possible U(1) charges that the black holes can carry, while the five dimensional solutions in [2] can carry three charges. Each of the different charges

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arises from a different species of giant graviton. In all cases, the single charge BPS solution has a horizon that coincides with the curvature singularity and adding some energy creates a solution with finite gravitational entropy.

As is well known, $\mathrm{AdS}_4 \times \mathrm{S}^7$, $\mathrm{AdS}_5 \times \mathrm{S}^5$ and $\mathrm{AdS}_7 \times \mathrm{S}^4$ arise in string theory as the near horizon geometries of stacks of M2, D3 or M5-branes. Typically, we take the near-horizon limit of a flat brane and find the Poincaré patch of AdS_k, and a duality follows with the low energy world-volume CFT of the corresponding planar $k-2$ brane. The charged black holes studied here are in global AdS space, so we do not have such a near-horizon construction.[1] Nevertheless, these spaces are dual to a $k-2$ brane CFT on a sphere. Hence we will speak of N spherical "background" branes creating the spacetime. As described above, the single charge black hole solutions of these theories can be interpreted as condensates of giant gravitons which are themselves spherical branes moving on the sphere factor of $\mathrm{AdS}_k \times \mathrm{S}^l$. The giants of AdS_4, AdS_5 and AdS_7 are spherical M5, D3 and M2 branes respectively. Near any one of these branes, the geometry should locally be $\mathrm{AdS}_l \times \mathrm{S}^k$. When the number of giants is very large, the background spacetime should be dominated by the presence of the giants rather than the presence of background branes. This leads to an intriguing hypothesis: *When the number of condensed giants N_g is much greater than the number of background branes N, the system has an equivalent description in terms of N giant gravitons condensed in a background created by N_g branes .*

In Section 4 we find evidence for such a correspondence by examining the thermodynamics of near-BPS, single charge black holes of $\mathrm{AdS}_k \times \mathrm{S}^l$. We measure entropies and temperatures in terms of the number of condensed giants, and find an exact match between the large charge black holes in $\mathrm{AdS}_k \times \mathrm{S}^l$ and the small charge black holes in $\mathrm{AdS}_l \times \mathrm{S}^k$ when the number of giants is exchanged with the number of background branes while holding energies and the AdS scale fixed. Implications of our findings and directions forward are discussed in Section 5.

2 Four dimensions

The Kaluza-Klein compactification of M-theory on S^7 can be truncated consistently to $SO(8)$ gauged $\mathcal{N} = 8$ supergravity in four dimensions. We are interested in black holes charged under the maximal Abelian subgroup $U(1)^4$. There is a truncation of the full $\mathcal{N} = 8$, $SO(8)$ theory to a $U(1)^4$, $\mathcal{N} = 2$ theory, for which the bosonic fields are the metric $g_{\mu\nu}$, four $U(1)$ gauge fields A_i, three scalars and three pseudo-scalars (the pseudo-scalars will be set to zero in this note). This theory admits four-charge

[1] However, see [5] for global AdS_3 arising from the near-horizon limit of the spinning D1-D5 string. It would be interesting if a similar construction could be carried out in higher dimensions from the near horizon limit of spinning branes.

AdS black hole solutions, given by [6]:

$$\begin{aligned} ds_4^2 &= -(H_1H_2H_3H_4)^{-1/2} f\, dt^2 + (H_1H_2H_3H_4)^{1/2}(f^{-1}dr^2 + r^2 d\Omega_2^2), \\ X_i &= H_i^{-1}(H_1H_2H_3H_4)^{1/4}, \qquad A^i = \frac{\tilde{q}_i}{r+q_i}\, dt, \qquad (i = 1\cdots 4) \end{aligned} \tag{1}$$

where

$$f = 1 - \frac{\mu}{r} + \frac{r^2}{L_4^2}(H_1H_2H_3H_4), \qquad H_i = 1 + \frac{q_i}{r}.$$

The four X_i satisfy the relation $X_1X_2X_3X_4 = 1$, parameterizing three physical scalars, while μ is a SUSY breaking parameter. The BPS solution occurs when $\mu = 0$. The physical $U(1)$ charges $\tilde{q}_i$ are given in terms of q_i as

$$\tilde{q}_i = \sqrt{q_i(\mu + q_i)}\,. \tag{2}$$

The mass of this black hole is:

$$M_4 = \frac{1}{4G_4}(2\mu + \sum_{i=1}^{4} q_i) \equiv \frac{1}{4G_4}\sum_{i=1}^{4} q_i + 2\,\delta M_4\,, \tag{3}$$

where $2\delta M_4$ measures the deviation from the BPS limit. Black holes carrying a charge under one of the four $U(1)$'s have a curvature singularity surrounded by horizon, which shrinks to zero area as $\mu \to 0$. The multi-charge solutions are qualitatively different. They have a critical value, $\mu = \mu_{crit}$, below which the horizon disappears giving a nakedly singular space. For $\mu > \mu_{crit}$, there is a regular black hole horizon with finite entropy and a non-zero temperature. As $\mu \to \mu_{crit}$ from above, the horizon approaches a finite limiting area. This critical black hole has zero temperature, but finite entropy [7].

Lifting to 11D supergravity: The general Kaluza-Klein ansatz that lifts any solution of 4D $\mathcal{N} = 2, U(1)^4$ gauged SUGRA to a solution of 11D supergravity was given in [4]. The charged black hole solution (1) lifts to

$$\begin{aligned} ds_{11}^2 &= \Delta^{2/3}\Big((H_1H_2H_3H_4)^{-1} f\, dt^2 + (f^{-1}dr^2 + r^2 d\Omega_2^2)\Big) \\ &\quad + 4\Delta^{-1/3}\sum_i H_i\Big(L_4^2\, d\mu_i^2 + \mu_i^2\,(L_4\, d\phi_i + \frac{\tilde{q}_i}{r+q_i}dt)^2\Big)\,, \end{aligned} \tag{4}$$

where $\sum \mu_i^2 = 1$, $\Delta = (H_1H_2H_3H_4)\sum_{i=1}^4(\mu_i^2/H_i)$, and $d\Omega_2^2 = \sin^2\alpha_1 d\alpha_1 d\alpha_2$ is the volume element on a unit two-sphere. The solution is asymptotically $\mathrm{AdS}_4 \times \mathrm{S}^7$ where the AdS and sphere length scales are L_4 and $2L_4$ respectively. The sphere is parameterized by ϕ_i and μ_i as

$$d\Omega_7^2 = \sum_i (d\mu_i^2 + \mu_i^2 d\phi_i^2)\,, \tag{5}$$

and the four μ_i are expressed in terms of three angles as

$$\mu_1 = \cos\theta_1,\ \ \mu_2 = \sin\theta_1\cos\theta_2,\ \ \mu_3 = \sin\theta_1\sin\theta_2\cos\theta_3,\ \text{ and }\ \mu_4 = \sin\theta_1\sin\theta_2\sin\theta_3.$$

The lift of the black hole also has a four form field strength $F^{(4)} = dB^{(3)}$ with

$$B^{(3)} = -\frac{r^3}{L_4}\Delta\, dt\wedge d\Omega_2^2 - L_4\sum_{i=1}^{4} q_i\mu_i^2(L_4\, d\phi_i - dt)\wedge d\Omega_2^2\,. \tag{6}$$

Interpretation as condensed giants: The lifted 11 D black hole solutions have curvature singularities localized in AdS_4 and distributed all over S^7. These singularities are even naked in the multi-charge cases for small μ. In the analogous $\mathrm{AdS}_5\times\mathrm{S}^5$ solutions, Myers and Tafjord [2] argued that the singularity could be understood as a condensate of giant gravitons, by showing that the 5-form flux near the singularity in their solution was consistent with presence of a distribution of spherical D3-branes on S^5. Then, using the charge-mass relation of BPS giant gravitons they showed that the BPS solutions have an ADM mass that is also consistent with a source that is a condensate of giant gravitons [2].

In our case, the relevant brane source will be a distribution of giant gravitons on S^7. These are spherical M5 branes occupying an S^5 of the S^7 and carrying angular momentum along one direction of the sphere [3]. We will start with the BPS case ($\mu = 0$) and a single non-zero charge (q_1) and then state the results for the general case.

Being spherical branes, our giants correspond to M5 brane dipoles. They locally excite the four-form field strength which can be detected by integrating the dual four-form on a surface enclosing a part of the M5-sphere. A closed surface transverse to the M5-brane spans the angular coordinates of AdS_4 and the directions on S^7 that are transverse to the M5-brane. The relevant four-form component is therefore

$$F^{(4)}_{\theta_1\phi_1\alpha_1\alpha_2} = 4q_1L_4^2\sin\theta_1\cos\theta_1\sin\alpha_1\,. \tag{7}$$

Integrating this form over the angular coordinates transverse to the giant gravitons ($\alpha_{1,2}$ in AdS and ϕ_1 and θ_1 on the sphere) at any fixed r and t gives a net flux which is proportional to the number of enclosed giants. Following [2, 3], giant gravitons exciting these four-form components are moving in the ϕ direction of S^7 and localized along the θ direction. We can express this number in terms of 11d Planck length l_p and N which counts the number of units of background 4-form flux that are present independently of the giant gravitons, or equivalently the number of M2-branes whose near-horizon limit yields $\mathrm{AdS}_4\times\mathrm{S}^7$ with AdS length scale $L_4 = l_p(\frac{1}{2}\pi^2N)^{1/6}$. Then, with our conventions,

$$16\pi G_{11}T_5n_1 = \int d\theta_1 d\phi_1 d\alpha_1 d\alpha_2\, F_{\theta_1\phi_1\alpha_1\alpha_2}\,, \tag{8}$$

where $G_{11} = 16\pi^7 l_p^9$ and $T_5 = \frac{2\pi}{(2\pi l_p)^6}$ is the M5 brane tension. By dropping the integration over θ_1, we obtain the distribution of giant gravitons in θ_1:

$$\frac{dn_1}{d\theta_1} = \frac{N^{1/2}}{8\sqrt{2}\pi^2 L_4^3} \int F^{(4)}_{\theta_1\phi_1\alpha_1\alpha_2} d\phi_1 d^2\alpha = 2N^{1/2} \frac{q_1}{\sqrt{2}L_4} \sin 2\theta_1 \,. \tag{9}$$

Integrating over θ_1 gives the total number of giants,

$$n_1 = \int_0^{\pi/2} d\theta_1 \frac{dn_1}{d\theta_1} = \sqrt{2N} \frac{q_1}{L_4} \,. \tag{10}$$

Treating a single giant graviton as a probe in a background $\mathrm{AdS}_4 \times \mathrm{S}^7$ geometry, one finds that a giant located at θ_1 has a radius $L_4 \sin\theta_1$ and carries an angular momentum along the ϕ_1 direction equal to $N \sin^4\theta_1$ [3]. Using this relation, we expect that the total angular momentum of the condensate of giants (9) is

$$P_{\phi_1} = \int_0^{\pi/2} d\theta_1 \frac{dn_1}{d\theta_1} N \sin^4\theta_1 = \frac{2N^{3/2}}{3\sqrt{2}} \frac{q_1}{L_4} \,. \tag{11}$$

Likewise, using the energy-momentum relation of BPS giant gravitons, we expect that a condensate with distribution (9) has a total energy

$$E = \frac{P_{\phi_1}}{2L} = \frac{N^{3/2}}{3\sqrt{2}} \frac{q_1}{L_4^2} \,. \tag{12}$$

This agrees beautifully with the ADM mass (3)

$$M_4 = \frac{1}{4G_4} q_1 = \frac{N^{3/2}}{3\sqrt{2}} \frac{q_1}{L_4^2} \,, \tag{13}$$

where we have used $G_4 = G_{11}/\mathrm{Vol}(S^7) = \frac{3G_{11}}{128\pi^4 L_4^7}$.

We can extend this analysis to the multi-charge black hole solutions with arbitrary q_i. The generic BPS solution is nakedly singular. The singularity arises from condensation of sets of giant gravitons, separately moving along the different ϕ_i. It is convenient to use the radius of the giant graviton moving along ϕ_i as a coordinate $\rho_i = 2L_4\sqrt{1-\mu_i^2}$. Then, an analysis like the one given above shows that the distribution of each set of giant gravitons is

$$\frac{dn_i}{d\rho_i} = 2\sqrt{2N} \frac{q_i}{L_4^3} \rho_i \,. \tag{14}$$

The giant graviton of radius ρ_i carries an angular momentum $N\rho_i^4/L_4^4$. The total angular momentum carried by each set of giant gravitons is

$$P_i = \frac{2N^{3/2}}{3\sqrt{2}} \frac{q_i}{L_4} \,. \tag{15}$$

Likewise, the total energy of the giant gravitons $E = \sum P_i/2L_4$ is in agreement with the ADM mass of the multi-charge black hole.

Beyond BPS: So far we have considered the BPS solutions with $\mu = 0$. As a result, we were able to compute energy of a condensate of probe giant gravitons following the distribution (9) and this exactly matched the gravitational mass of the fully back-reacted spacetime solution. Repeating the analysis in the non-supersymmetric case when $\mu > 0$, we find that the net giant number density, the net number of giants and their net momentum are simply given by replacing q_i in (9), (10) and (11) by the physical charge $\tilde{q}_i$ (2) of the non-extremal solution. However, replacing q_i by $\tilde{q}_i$ in the giant energy formula (12) does not reproduce the spacetime mass (3) because the non-extremality can produce additional fluctuations of the giant gravitons as well as giant-anti-giant pairs at any fixed net momentum (11).

3 Seven dimensions

In an analysis parallel to the one above, we can consider the $\mathcal{N} = 4$, $SO(5)$ gauged supergravity in 7 dimensions, arising from dimensional reduction and consistent truncation of 11d SUGRA on S^4. This theory can be further truncated to $\mathcal{N} = 2$, $U(1)^2$ gauged SUGRA, where the only bosonic fields retained are the metric, two gauge potentials and two scalars [4].[2] The charged black hole solutions in this theory are

$$\begin{aligned} ds_7^2 &= -(H_1 H_2)^{-4/5} f\, dt^2 + (H_1 H_2)^{1/5} (f^{-1} dr^2 + r^2 d\Omega_5^2)\,, \\ X_i &= H_i^{-1} (H_1 H_2)^{2/5}\,, \qquad A^i = \frac{\tilde{q}_i}{r^4 + q_i}\, dt\,, \end{aligned} \tag{16}$$

and

$$f = 1 - \frac{\mu}{r^4} + \frac{r^2}{L_7^2}(H_1 H_2), \qquad H_i = 1 + \frac{q_i}{r^4}, \qquad i = 1, 2. \tag{17}$$

Here μ is a SUSY-breaking parameter and the $\tilde{q}_i$ are defined as in (2). The mass of this solution is

$$M_7 = \frac{\pi^2}{4G_7}(\frac{5\mu}{4} + q_1 + q_2) = \frac{\pi^2}{4G_7}(q_1 + q_2) + 5\delta M_7\,, \tag{18}$$

where δM_7 measures the deviation from the BPS limit. Using the general Kaluza-Klein ansatz to lift to a solution of 11d SUGRA [4], the charged black hole solution (16) becomes

$$\begin{aligned} ds_{11}^2 &= \Delta^{1/3} \Big((H_1\, H_2)^{-1} f\, dt^2 + (f^{-1} dr^2 + r^2 d\Omega_5^2) \Big) \\ &\quad + \frac{1}{4} \Delta^{-2/3} \Big[d\mu_0^2 + \sum_{i=1}^{2} H_i \Big(L_7^2\, d\mu_i^2 + \mu_i^2 (L_7\, d\phi_i + \frac{\tilde{q}_i}{r^4 + q_i} dt)^2 \Big) \Big]\,, \end{aligned}$$

[2] This is not in general a consistent truncation but is so for solutions of the form considered here [4].

where

$$\Delta \equiv (H_1 H_2)\Big(\mu_0^2 + \sum_{i=1}^{2} \frac{\mu_i^2}{H_i}\Big), \qquad \mu_0 \equiv \sin\theta_1 \sin\theta_2, \qquad \mu_1 \equiv \cos\theta_1, \qquad \mu_2 \equiv \sin\theta_1\, \cos\theta_2\,, \tag{19}$$

and $L_7^2 = 4 l_p^2 (\pi N)^{2/3}$. This solution is asymptotically $\mathrm{AdS}_7 \times \mathrm{S}^4$ with scales L_7 and $L_7/2$ respectively. N counts the number of units of background 7-form flux, or equivalently, the number of M5-branes whose near-horizon limit is $\mathrm{AdS}_7 \times \mathrm{S}^4$ with AdS_7 scale L_7. First consider the BPS solution ($\mu = 0$). Following the reasoning in [2] and the previous section, the relevant 7-form component is

$$F^{(7)}_{\rho_i \phi_i \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5} = 4\, q_i\, \rho_i\, \sin^4 \alpha_1\, \sin^3 \alpha_2\, \sin^2 \alpha_3\, \sin \alpha_4\,, \tag{20}$$

where $\rho_i \equiv \frac{L_7}{2}\sqrt{1-\mu_i^2}$. The density of giant gravitons is

$$\frac{dn_i}{d\rho_i} = \frac{1}{16\pi G_{11} T_2} \int F^{(7)}_{\rho_i \phi_i \alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5} d\phi_i d^5\alpha = 8N^2 \frac{q_i}{L_7^6} \rho_i\,. \tag{21}$$

The total number of giant gravitons in each set is

$$n_i = \int_0^{L/2} d\rho_i 8N^2 \frac{q_i}{L_7^6}\rho_i = N^2 \frac{q_i}{L_7^4}\,. \tag{22}$$

Giant gravitons of radius ρ_i carry angular momentum $\frac{2N}{L_7}\rho_i$ [3] and therefore the total angular momentum carried by each set of giant gravitons is

$$P_{\phi_i} = \int_0^{L/2} d\rho_i\, \frac{dn_1}{d\rho_i} \frac{N}{L_7} \rho_i = \frac{2N^3}{3} \frac{q_i}{L_7^4}\,. \tag{23}$$

Likewise, using the energy-momentum relation for giants, the total energy of the condensate of giants is

$$E = \frac{P_{\phi_1} + P_{\phi_2}}{L_7/2} = \frac{4N^3}{3} \frac{q_1 + q_2}{L_7^5}\,. \tag{24}$$

As before, these energy and momentum calculations are performed while treating giant gravitons as probes in a background $\mathrm{AdS}_7 \times \mathrm{S}^4$ geometry. Nevertheless, the total energy agrees exactly with the spacetime mass of the solution (18)

$$M_7 = \frac{\pi^2}{4G_7}(q_1 + q_2) = \frac{4N^3}{3} \frac{q_1 + q_2}{L_7^5}\,. \tag{25}$$

where we have used $G_7 = 6G_{11}/(\pi^2 L_7^4)$. In contrast, the non-supersymmetric solutions with $\mu > 0$ have a net condensate of giants in which q_i in (21) and (23) is replaced by the physical charge $\tilde{q}_i$ defined as in (2). A similar replacement in the giant energy formula (24) will not reproduce the mass (18) because at positive μ additional excitations of the giants as well as brane-anti-brane pairs may be present.

4 Entropy

Above we have generalized the calculation of Myers and Tafjord [2] in 5d to interpret certain charged black holes and curvature singularities of 4d and 7d gauged supergravity as condensates of giant gravitons. Giant gravitons are themselves spherical M2, D3 and M5 branes, and so we expect that after including gravitational backreaction the geometry very close to the surface of a giant should be locally $\mathrm{AdS}_l \times \mathrm{S}^k$. In particular, near the M5-brane giants of the $\mathrm{AdS}_4 \times \mathrm{S}^7$ space, the geometry should be locally $\mathrm{AdS}_7 \times \mathrm{S}^4$. Likewise, near the M2-brane giants of $\mathrm{AdS}_7 \times \mathrm{S}^4$ the local geometry should be $\mathrm{AdS}_4 \times \mathrm{S}^7$. Finally, near the D3-brane giants of $\mathrm{AdS}_5 \times \mathrm{S}^5$ the local geometry should again be $\mathrm{AdS}_5 \times \mathrm{S}^5$, but the AdS scale should be determined by N_g, the number of giants, rather than N, the number of background branes. This suggests a simple correspondence principle that should hold when a single species of giant is condensed: *When the number of giants in* $\mathrm{AdS}_k \times \mathrm{S}^l$ *is sufficiently large, the physics can be equivalently described by a dual* $\mathrm{AdS}_l \times \mathrm{S}^k$ *in which the giants and the background branes have exchanged roles.*

If this correspondence is correct, the thermodynamics of the large charge black holes of $\mathrm{AdS}_k \times \mathrm{S}^l$ and the small charge black holes of $\mathrm{AdS}_l \times \mathrm{S}^k$ should be equivalent. In the near-BPS limit we expect it to imply equality of black hole entropies and temperatures. Below, we test this by expressing the entropy and temperature of near-BPS, single charge black holes of $\mathrm{AdS}_k \times \mathrm{S}^l$ in terms of the number of condensed giants N_g, the number of background branes N and the energy above extremality δM.

$\mathbf{AdS_5 \times S^5}$: The previous sections have discussed the charged black hole solutions of AdS_7 and AdS_4 but here we begin by analyzing the entropy of the AdS_5 black holes whose interpretation in terms of giant gravitons was given in [2]. In detail, the $\mathcal{N} = 8$, $SO(6)$ gauged SUGRA arising from consistent truncation of 10d, IIB SUGRA compactified on S^5 has a further truncation to 5d $\mathcal{N} = 2$, $U(1)^3$ gauged SUGRA. The black hole solutions of this theory are [8]

$$\begin{aligned} ds_5^2 &= -(H_1H_2H_3)^{-2/3} f\, dt^2 + (H_1H_2H_3)^{1/3}(f^{-1}dr^2 + r^2 d\Omega_3^2), \\ X_i &= H_i^{-1}(H_1H_2H_3)^{1/3}, \qquad A^i = \frac{\tilde{q}_i}{r+q_i}\, dt, \qquad (i = 1 \cdots 4) \end{aligned} \tag{26}$$

where

$$f = 1 - \frac{\mu}{r^2} + \frac{r^2}{L_5^2}(H_1H_2H_3), \qquad H_i = 1 + \frac{q_i}{r^2}. \tag{27}$$

The solution has a mass

$$M_5 = \frac{\pi}{4G_5}(\frac{3}{2}\pi\mu + \sum q_i) = \frac{\pi}{4G_5}\sum q_i + 3\,\delta M_5\,. \tag{28}$$

Here $3\delta M_5$ measures the additional mass over the BPS limit produced by turning on μ. In the single charge BPS case ($\mu = 0$) Myers and Tafjord showed that the singularity contains

$$N_g = N\,\frac{q}{L_5^2} \tag{29}$$

giants where $L_5^4 = 4\pi g_s N l_s^4$ and N is the number of background units of 5-form flux, or equivalently the number of 3-branes whose near-horizon limit gives AdS_5 with scale size L_5. Beyond the BPS limit ($\mu > 0$), the number of giants is given by replacing q in (29) by the physical charge $\tilde{q}$ as in (2). The horizon of the single charge black holes occurs at

$$r_h = \frac{1}{2}\sqrt{-2(L_5^2+q) + 2\sqrt{(L_5^2+q)^2 + 4\mu L_5^2}}\,. \tag{30}$$

We will consider near-BPS limits in which $\mu \ll q$ and compute the horizon entropy in the two cases $q \ll L_5^2$ and $q \gg L_5^2$, or equivalently $N_g \ll N$ and $N_g \gg N$. Gravitational entropy of a 5d horizon is given by

$$S = \frac{A}{4G_5} = \frac{\Omega_3}{4G_5} r_h^2 \sqrt{r_h^2 + q}\,, \tag{31}$$

where r_h is the location of the horizon, $G_5 = \frac{G_{10}}{\pi^3 L_5^5}$ and $G_{10} = 8\pi^6 g_s^2 l_s^8$ are the 5d and 10d Newton constants.[3] In terms of the number of giants we find that

Limits	r_h	$S = \frac{A}{4G_4}$
$\mu \ll q,\ N_g \ll N$	$\sqrt{\mu}$	$S = 4\pi(L_5\,\delta M_5)\sqrt{\frac{N_g}{N}}$
$\mu \ll q,\ N_g \gg N$	$r_h = L_5\,\sqrt{\mu/q}$	$S = 4\pi(L_5\,\delta M_5)\sqrt{\frac{N}{N_g}}$

(32)

Notice that in the small and large charge limits, the entropy formula interchanges the roles of the giant gravitons (N_g) and the D3-branes (N) whose near-horizon created the background geometry, if we hold L_5 and δM_5 fixed.

$AdS_4 \times S^7$: We will study single charge 4d black holes (1) for which the horizon occurs when

$$f = 1 - \frac{\mu}{r} + \frac{qr}{L_4^2} + \frac{r^2}{L_4^2} = 0\,. \tag{33}$$

As above we will compare the entropy of small charge ($q \ll L_4$) and large charge ($q \gg L_4$) black holes. From (10), the number of giants N_g satisfies $N_g \ll \sqrt{N}$ and $N_g \gg \sqrt{N}$ in these cases. We will study the near-BPS solutions for which $\mu \ll q$, but in the large charge case will need to separately consider the regimes $\mu \ll L_4^2/q$ ($\frac{\mu}{L_4} \ll \frac{\sqrt{N}}{N_g}$) and $L_4^2/q \ll \mu \ll q$ ($\frac{\sqrt{N}}{N_g} \ll \frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}}$). The gravitational entropy is

$$S = \frac{A}{4G_4} = \frac{\Omega_2}{4G_4}\, r_h^2 \sqrt{1 + \frac{q}{r_h}}\,, \tag{34}$$

[3] $\Omega_D = \frac{2\pi^{(d+1)/2}}{\Gamma[(d+1)/2]}$ is the volume of the unit D-sphere.

where r_h is the location of the horizon. Then, using $G_4 = G_{11}/\mathrm{Vol}(\mathrm{S}^7)$ and G_{11} as in Sec. 2, we obtain,

Limits	r_h	$S = \frac{A}{4G_4}$
$\frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}},\ N_g \ll \sqrt{N}$	μ	$\sqrt{48\pi^2}\,(L_4\,\delta M_4)^{3/2}\,\frac{\sqrt{N_g}}{N}$
$\frac{\mu}{L_4} \ll \frac{\sqrt{N}}{N_g},\ N_g \gg \sqrt{N}$	μ	$\sqrt{48\pi^2}\,(L_4\,\delta M_4)^{3/2}\,\frac{\sqrt{N_g}}{N}$
$\frac{\sqrt{N}}{N_g} \ll \frac{\mu}{L_4} \ll \frac{N_g}{\sqrt{N}},\ N_g \gg \sqrt{N}$	$L_4\sqrt{\mu/q}$	$4\pi\frac{1}{3^{1/4}}\,(L_4\,\delta M_4)^{3/4}\,\frac{\sqrt{N}}{N_g^{1/4}}$

(35)

Here δM_4 is a measure of energy over the BPS limit as in (3). Unlike the AdS_5 case the entropies for the small and large charge cases do not seem to be related to each other. However, we will see a remarkable fact below – the large charge AdS_4 black hole entropy is related to the small charge AdS_7 entropy and vice versa.

$\mathbf{AdS_7 \times S^4}$: The horizon of the single charge 7d black hole (16) occurs when

$$1 - \frac{\mu}{r^4} + \frac{r^2}{L_7^2} + \frac{q}{r^2 L_7^2} = 0\,. \tag{36}$$

We will consider small charge ($q \ll L_7^4$) and large charge ($q \gg L_7^4$) limits, which imply $N_g \ll N^2$ and $N_g \gg N^2$ respectively from(22). As above, we will study the near-BPS ($\mu \ll q$) limit, but in the small charge case we must separately consider the cases $\left(q/L_7^2\right)^2 \ll \mu \ll q$ $\left(\frac{N_g^2}{N^4} \ll \frac{\mu}{L_7^4} \ll \frac{N_g}{N^2}\right)$ and $\mu \ll (q/L_7^2)^2$ $\left(\frac{\mu}{L_7^4} \ll \frac{N_g^2}{N^4}\right)$. The gravitational entropy is given by

$$S = \frac{A}{4G_7} = \frac{\Omega_5}{4G_7}\sqrt{1 + \frac{q}{r_h^4}}\,r_h^5\,. \tag{37}$$

Using $G_7 = G_{11}/\mathrm{Vol}(\mathrm{S}^4)$ and G_{11} as in Sec. 3,

Limits	r_h	$S = \frac{A}{4G_7}$
$\frac{\mu}{L_7^4} \ll \frac{N_g}{N^2},\ N_g \gg N^2$	$L_7\sqrt{\mu/q}$	$\sqrt{48\pi^2}\,(L_7\,\delta M_7)^{3/2}\,\frac{\sqrt{N}}{N_g}$
$\frac{\mu}{L_7^4} \ll \frac{N_g^2}{N^4},\ N_g \ll N^2$	$L_7\sqrt{\mu/q}$	$\sqrt{48\pi^2}\,(L_7\,\delta M_7)^{3/2}\,\frac{\sqrt{N}}{N_g}$
$\frac{N_g^2}{N^4} \ll \frac{\mu}{L_7^4} \ll \frac{N_g}{N^2},\ N_g \ll N^2$	$\mu^{1/4}$	$4\pi\frac{1}{3^{1/4}}\,(L_7\,\delta M_7)^{3/4}\,\frac{\sqrt{N_g}}{N^{1/4}}$

(38)

Here δM_7 is a measure of energy over the BPS limit as in (18).

4.1 A thermodynamic correspondence

The entropy formulae derived above were necessarily functions of δM, the energy above extremality. However, since entropy is itself dimensionless, this energy always appears in the dimensionless combination $L\,\delta M$, where L is the AdS scale. The appearance of this combination, which is natural from the point of the gravitational solution, also has a natural interpretation from the microscopic point of view. Das, Jevicki and Mathur have shown that a giant graviton has a discrete spectrum that is determined by the AdS scale L and is *independent* of all other length scales including the radius of the giant [9]. This remarkable fact, coupled with the appearance of $L\,\delta M$ in the above entropy formulae, suggests that the thermodynamics of the charged black holes studied here can be microscopically explained in terms of the fluctuations of giant gravitons that make up the singularity. In effect, the giant gravitons appear to play a role for our black holes analogous to the role of the D1-D5 string in the classic Strominger-Vafa analysis of black hole entropy in string theory [10].

Above we argued that when the number of giants in the solution is large there ought to be a correspondence exchanging the roles of the giant gravitons and the background branes creating the AdS spacetime. Essentially, this should relate the entropies of large charge black holes in $\mathrm{AdS}_l \times \mathrm{S}^k$ and small charge black holes in $\mathrm{AdS}_k \times \mathrm{S}^l$. The results presented in (32), (35) and (38) are a striking confirmation of this fact. In AdS_5 the entropy at large N_g is obtained from the small N_g result by exchanging N and N_g while holding the dimensionless energy $L_5\,\delta M_5$ fixed. Similarly, the large N_g entropy in AdS_4 reproduces the small N_g entropy in AdS_7 if we hold the dimensionless energy fixed ($L_4\,\delta M_4 = L_7\,\delta M_7$) while exchanging N_g and N. The numerical factors match exactly, as do the limits on the supersymmetry breaking parameter μ.

We can derive the temperature of our solutions from the thermodynamic relation $\beta = 1/T = dS/dE$. By requiring that the temperatures match between corresponding solutions in $\mathrm{AdS}_l \times \mathrm{S}^k$ and $\mathrm{AdS}_k \times \mathrm{S}^l$ we find that while exchanging N and N_g we should separately hold δM and the supergravity length scale L constant. Since L can be expressed in terms of the number of background branes and the fundamental length scales $l_5 = g_s^{1/4} l_s$ or the Planck length l_p, this requires us to rescale l_5 or l_p when N_g and N are exchanged. In particular, $N_g \gg N$ solutions in AdS_5 with a given value of $l_5 = g_s^{1/4} l_s$ are related to the $N_g \ll N$ solutions with $\tilde{l}_5 = l_5(N/N_g) \ll l_5$. Similarly, the $N_g \gg \sqrt{N}$ solutions in AdS_4 with a given Planck length l_p are related to $N_g \ll N^2$ spaces in AdS_7 with $\tilde{l}_p = l_p(1/2)^{7/6}(N/N_g^2)^{1/6} \ll l_p$. Finally, the $N_g \gg N^2$ solutions in AdS_7 with a given l_p are related to $N_g \ll \sqrt{N}$ black holes in AdS_4 with $\tilde{l}_p = l_p(2)^{7/6}(N^2/N_g)^{1/6} \ll l_p$. In all cases, the new fundamental length scale is much smaller than the original scale. While this will be important for a quantum mechanical analysis of our proposed correspondence, it is not relevant for the supergravity analysis of this article since only the scale L appears in the spacetime solutions.

5 Discussion: A correspondence principle?

We have presented evidence from supergravity that there is a correspondence between the physics of certain large charge black holes in $\mathrm{AdS}_l \times \mathrm{S}^k$ and small charge black holes in $\mathrm{AdS}_k \times \mathrm{S}^l$. It would be interesting to extend the correspondence that we suggest to a full quantum mechanical duality.

To this end, it is important to understand how the charged black holes discussed here are represented in the CFT dual to AdS space. In [11, 12] it was shown that non-normalizable bulk modes (boundary conditions) map to CFT couplings, while normalizable supergravity lumps give rise to VEVs for CFT operators. The operators relevant to condensation of giant gravitons were identified in [13] as subdeterminants of products of CFT fields. Presumably, the black holes we have studied are related to superselection sectors of the CFT in which these operators have VEVs.

The CFT state corresponding to large N_g would not be well-approximated by the planar limit. (See [13] for a discussion of the combinatorial explosion in perturbative calculations involving giant gravitons.) In the bulk spacetime non-planar diagrams correspond to string loop corrections. The correspondence we are proposing could be understood in AdS_5 as stating that these loop corrections re-sum to give another description in which the $SU(N)$ gauge symmetry of the dual CFT is exchanged with an $SU(N_g)$ symmetry arising from the condensed giants.

A brief glance at the entropy formulae in (32), (35) and (38) and the relation $1/T = dS/dE$ shows that the black holes that we are examining have unusual thermodynamic relations. For example, in the limits we examined in (32) the temperature is independent of the energy. In fact, some of the solutions we have considered here have thermodynamic, and possibly dynamical, instabilities [7, 14]. We expect that such solutions with large charges in $\mathrm{AdS}_k \times \mathrm{S}^l$ will correspond to unstable small charge solutions in $\mathrm{AdS}_l \times \mathrm{S}^k$ and that the instabilities can be related to each other. Understanding this will be important for the goal of relating various features of the gravitational solutions to the physics of the giant gravitons making up the singularities.

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